

Overhang: Corbelled Structural Systems

Corbelled structures haven't grabbed the attention of the architectural and structural community in the last century. They could be considered the poor relative in the family of structural systems. A couple of recent mathematical papers, however, reveal new and groundbreaking insights into the nature of corbelled structures as they have never been actualized in architecture. This paper documents research that imports this knowledge into the discipline of architecture for the first time.

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INTRODUCTION

This paper documents current research being done in the design of corbelled dry-stacked compressive structures.

Corbelled structures haven't grabbed the attention of the architectural and structural community in the last century. They could be considered the poor relative in the family of structural systems. A couple of recent mathematical papers, however, reveal new and groundbreaking insights into the nature of corbelled structures as they have never been actualized in architecture. This paper documents research that imports this new mathematical knowledge into the discipline of architecture, investigating the formal and spatial design implications of the mathematical findings.

Corbelled structures offer a couple of advantages over true arches: Firstly, they do not need formwork to be constructed. Secondly, they result in constructions free from tensile and lateral forces at the joints, making reinforcement unnecessary, which is interesting for a series of building materials. Concrete, for example, is a material with an extremely long potential lifespan. This potential longevity, however, is compromised by the relatively short lifespan of steel reinforcement. Reinforcement is needed to take-on the tensile forces in concrete structures. Concrete and steel are two materials with different properties, behaviors and lifespans. Combining these materials leads to a shortened lifespan of the reinforced building element. The increase in tensile strength that steel provides to concrete is cancelled out by a decrease in the durability of the structure. Reinforced concrete bridges, for example, have an average life span of mere 50 years.

Although made of very brittle materials, most of the oldest surviving structures, like the Pantheon in Rome (built between approx. 110 and 128 CE¹) or the Treasury of Atreus in Mycenae (standing since approx. 1250 BCE²), do not have reinforcement. Historically, space was often spanned with corbelled, masonry structures. Corbelled structures survive longer than steel-reinforced concrete spans because they need more mass, larger redundancy and because the failure of a single element causes to a local failure in the overall structure instead

of a total collapse as in true arches. This long lifespan is, in part, the result of the high mass-to-void ratio in corbelled constructions. Considering the nearly two-thousand year lifespan of the Pantheon, the mass-to-void ratio of a non-reinforced concrete structure could be 38 times higher than contemporary reinforced concrete structure (with an average life span of 50 years) while still containing the same embodied energy. Additionally, the high mass-to-void ratio provides a high thermal mass, which would reduce the amount of energy needed for climate control.

This research investigates corbelled systems that avoid tensile forces altogether, creating purely compressive forces within the stacked assembly. Masonry structures that are constructed using dry-stacking, eliminating mortar, adhesive and steel reinforcement, have the potential to exploit the full longevity of a material such as concrete. Robust, long-lasting structures with the potential for re-use over centuries are our simplest, earliest precedents of sustainable buildings.

Until recently, it was thought that the harmonic stack allows the largest overhang of a fixed number of corbelled, stacked blocks as shown in Figure 1. Paterson, Peres, Thorup, Winkler and Zwick showed, however, that the largest overhang is actually achieved with totally irregular stack formations which cannot be described mathematically and can only be found through numerical approximation.³ Whereas the paper discussed two dimensional formations, this research investigates the spatial dimension of these stacking typologies and includes further parameters like the center of gravity, weight distribution and materiality. Beyond structural tests, this research extends to looking at the architectural possibilities and spatial applications of the three-dimensionalized sectional typologies discussed in “Maximum Overhang”.

This paper documents the following research, conducted empirically with scale models as well as digitally, using computer modeling and gravity simulation through the video game engine “Havok Physics”:

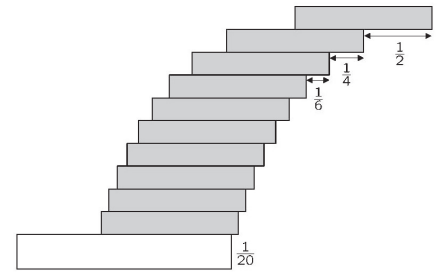
- stacked structures with varied material densities
- stacked structures with counterweights
- stacked structures with asymmetrical center points
- stacked structures with interlocking blocks

In the following section, this paper looks at recent research done by mathematicians in the area of stacked structures. Section three is a documentation of the completed research, and the last section describes the current stage of the research and outlines future steps.

MATHEMATICAL MODELS

Until quite recently it was believed that the harmonic stack afforded the maximum overhang with a fixed number of blocks.⁴ Several authors, however, showed that this is only true if the build-up technique is restricted to a single block per course. Once this restriction is lifted, larger overhangs can be achieved with a fixed number of blocks.⁵ Zwick and Paterson describe Sutton who showed an alternative three-block configuration with a larger overhang than a harmonic configuration. Zwick and Paterson also mention Hall who described alternative stack formations with more than 3 blocks. Zwick and Paterson believe that Hall’s formations are only optimal up to 19 blocks, but not for block numbers greater than 19. Hall’s configurations are “spinal stacks”. Because the terminology introduced by Zwick and Paterson is used throughout this paper, some explanation of the terms should be given at this point.

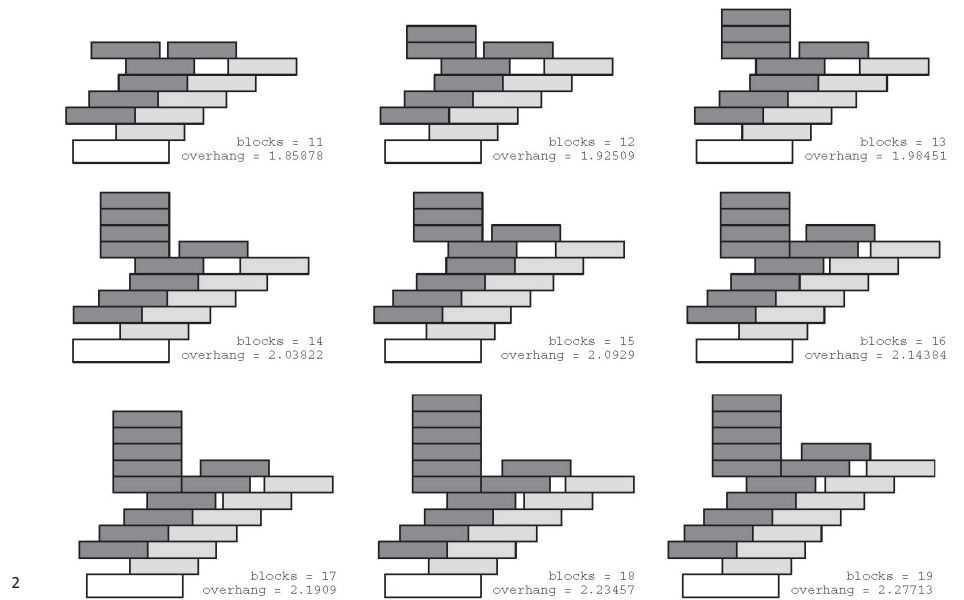
One key term is optimal stack. Optimal stack means a stack formation which allows for the maximum overhang possible with a specific number of blocks.



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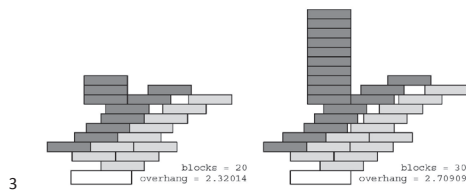
Figure 1: Harmonic stack. (Paterson & Zwick, 2009)

The principal block is the most overhanging block in a stack. Uri and Paterson describe the support set as the principal block and all blocks on which the principal block rests. All other blocks are called the balancing set. The balancing set serves as counterweight for the support set. Spinal stacks define stacks which have only one block per course in their support set as shown in Figure 2.



2

Zwick and Paterson show that optimal stacks with more than 19 blocks are not spinal⁶ and appear very irregular as shown in Figure 3. The light grey denotes the support set. Optimal stacks with 20 or more blocks have support sets with more than one block per course and are therefore not spinal. The 20 block optimal stack for example has 2 blocks in course 2 and 3 in its support set.



3

Until now, no algorithm has been found which describes the optimal formations which makes these geometries so fascinating. The optimal stack formations in “Overhang” are only approximations, but cannot be mathematically proven to be “optimal”.⁷

RESEARCH DOCUMENTATION

The findings of Zwick and Paterson suggest new branches of investigation into corbelled stacked structures as they question the dominance of the harmonic stack typology for corbelled structures and the cantilevering limits that come with it. Zwick and Paterson’s research was done within the context of mathematical research; there is no current research into the structural and architectural implications of the mathematical research findings. This research investigates the three-dimensional, architectural, formal, spatial and structural potential of irregular optimal and transformed harmonic stack formations.

In real-world applications, a number of parameters have to be introduced into the research done by the mathematicians. These parameters include materiality, density, friction and three-dimensionality or form. The research undertaken so far addresses some of these parameters and investigates their formal and spatial consequences. The research started in the form of two research seminars. The seminars included about 12 students each and were organized around research teams of 2–3 students. Each research team tackled an assigned area of investigation. The research teams were led by the author.

Figure 2: Optimal stack with 11 up to 19 blocks (Paterson & Zwick, 2009). Light gray blocks indicate the support set and dark gray blocks are the balancing set.

Figure 3: Optimal stack with 20 and 30 blocks (Paterson & Zwick, 2009). Light gray blocks indicate the support set and dark gray blocks are the balancing set. Optimal stack with 20 or more blocks are non-spinal.

In the following research documentation, a selection of work is organized within four categories: 1) three dimensional arrangement 2) density, 3) center of gravity and 4) block shape.

Some of the research projects are a mixture of the parameters listed, but have an emphasis on their assigned category. Investigations in “three dimensional arrangement” explore basic strategies of spatializing the flat stack sections in “Overhang”. The “density” category generates form and structure mainly through a specific distribution of weight in the overall structure. Projects in this category use materials with different densities. The project in the “center of gravity” category shifts the center of gravity inside the single blocks to generate form and structure. The projects in the form category transform the “block shape” to generate form and structure.

THREE DIMENSIONAL ARRANGEMENTS

The research done by Zwick and Paterson is solely done from a cross-sectional point of view. A first investigative step was to add a third dimension and to spatialize the stacks by Zwick and Paterson. Two basic strategies were tested: 1) extrusion of the stack perpendicular to the stack section and 2) rotation of the stack section around its principal block.

Extrusion

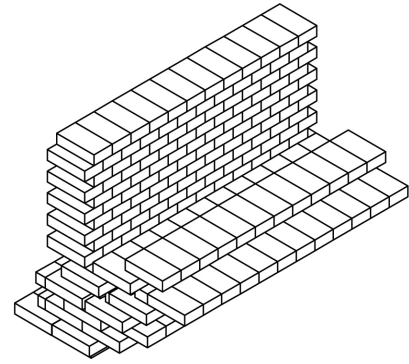
The simplest strategy to define a spatial volume and enclose space using a dry-stack is to extrude the corbelled stack. This requires the building-up of a number of stacks next to one another. In this arrangement, each stack is structurally independent from the ones built parallel. This arrangement creates a single spatial enclosure and experience, but structurally, the isolated overhangs are mostly independent. There is a slight increase in lateral stability because of the cumulative leaning effect of the stacks, but the extent of overhang does not change. Failure is also localized in this arrangement. If a stack collapses it does not compromise the structural integrity of the adjoining ones. The extruded stack could form a structural whole by simply shifting every second course by a half block width as shown in Figure 4.

Both interlocked and non-interlocked structures are able to enclose space without having gaps between the blocks. By mirroring the extrusion, a tunnel-like space is defined.

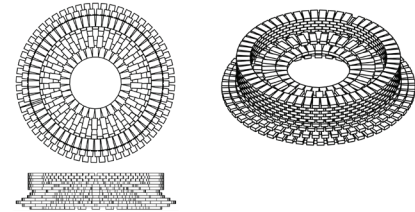
Rotation

The strategy here is to enclose space by rotating a stacked section around its principal block, creating a dome-like space. If the stacks do not interlock, the center cannot be covered, and an oculus stays open, as the principal blocks cannot share the central spot. In this arrangement, all stacks are structurally independent and failure would be local.

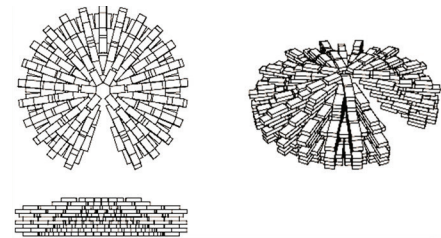
To integrate all blocks into a single structural unit in a rotated optimal stack typology, blocks at each course were arranged to fill each course circumference with the largest block number possible, by dividing the circumference with the block width as shown in Figure 5. As a consequence, the different courses overlap, creating a single structural entity. This strategy allows for a larger span compared to the non-interlocking version, as higher blocks are able to rest and distribute their weight onto lower support blocks. The top 8 principal blocks, for example, are ultimately resting on 52 base blocks. This arrangement also proved to be significantly more stable against lateral forces. All moment forces are confined within each block element. There is no outward lateral force created by this corbelled dome. Therefore, no buttresses are necessary. There is also no need for a bond between the blocks which differentiates it radically from a true dome. Unlike a true dome, the corbelled dome also does not need any formwork during construction. The principal blocks do not structurally depend on one another. This means that, unlike the Pantheon in Rome, there is no need for a pressure ring at the top. This typology shares the distribution of density logic of the Pantheon, and is still a purely corbelled dome as opposed to a true dome.



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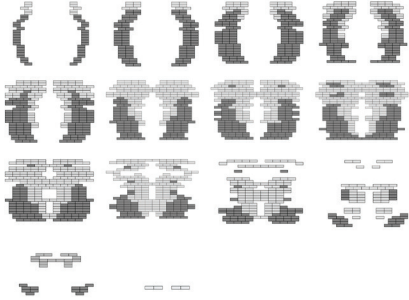
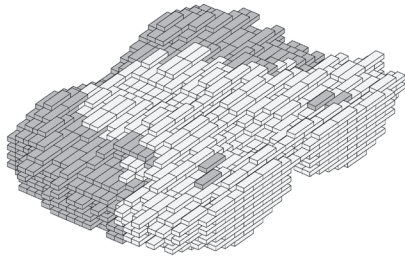


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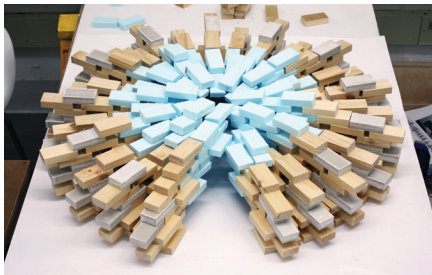
Figure 4: Optimal stack with blocks shifted every second course to create a bond between the section and forming a structural whole.

Figure 5: Rotated optimal stack typology in plan, axonometric and section views, with blocks overlapping from course to course. Gaps are diffusely distributed. If blocks are not tapered, little gaps are created between the blocks.

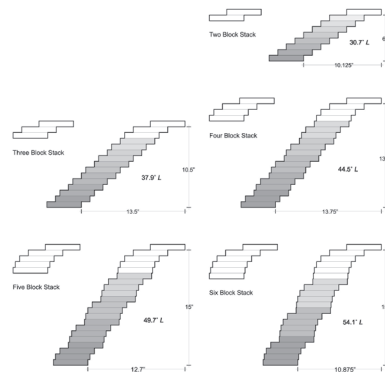
Figure 6: Rotated optimal stack typology in plan, axonometric and section views. The center, in this case, is covered. If blocks are not tapered, little gaps are created between the blocks.



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Figure 7: Courses 1 to 14. The gray shades indicate the two materials used: wood (dark gray) and styrofoam (light gray)

Figure 8: Simple rotated optimal stack, physical model test using 3 material densities, 640 blocks. Maximum overhang: 5 blocks.

Figure 9: Comparison between rates of density jumps. The density is reduced to a sixth of every previous density.

This arrangement creates gaps between the blocks which can filter exterior light. It is also simple to create an opening in the structure, by leaving a wedge section away, which would be difficult in a true dome structure. In comparison to a true dome, the lesser structural dependency of the individual parts allow for a potentially more open structure.

DENSITY

Investigations in the density category generate form and structure mainly through a deliberate and uneven distribution of weight in the overall structure. Through a distribution of different block densities, new spatial typologies are derived and the overhang is extended.

Extruded stack, variable density

This initial test in differentiated block densities uses two different material densities: wood and styrofoam. Using a simple extrusion of non-interlocked optimal stacks tests the formal effect of gradually shifting the balance of density, and therefore weight, from heavy to light blocks. Each section, in this test, contains 80 blocks. The first section contains 80 foam blocks. The second contains 76 foam blocks and 4 wood blocks. Each section continues to subtract 4 foam blocks and replace them with 4 wood blocks. After 20 iterations, the last section is made completely with wood blocks. Spatial articulation is created purely with the change in density distribution per stack as shown in Figure 7.

Simple rotated optimal stack, three densities

This test examines three different block densities: concrete, wood and polystyrene foam. The densities were applied in a rotated, interlocked optimal stack typology. This test shows how the typical balancing towers of the balancing set (see Figure 8) disappear. It creates a significantly slimmer section and overall appearance of the corbelled dome with a visually surprising overhang considering that it is a pure stack without any bonding between the blocks. Compared to a harmonic stack the overhang is extremely flat. Even compared to a true dome structure like the Pantheon, the overhang is far-reaching and flat, as can be seen in Figure 15. The far-reaching overhang is achieved by combining two strategies: First, all blocks are interlocked as described in the paragraph on the interlocked rotation. A higher block sits on more than one lower block distributing its weight and creating an interdependent structure. Also, higher blocks are lighter, having less weight to distribute in the first place. This arrangement rivals the cantilevering capacity of true arches with all the advantages of a corbelled arch described before: no formwork is needed during construction, lateral forces are controlled, many options for openings are possible, and there is a localized effect of any structural failure.

Exchange principal block with new harmonic stack with equal center of gravity and weight as principal block

This experiment pushes the idea of density distribution further. It sticks with the traditional typology of the harmonic stack and achieves greater overhang solely through a calibrated distribution of material density. The idea is that one could replace the principal block with a new harmonic stack with the equal weight and center of gravity position as the replaced principal block. By doing that, the cantilevering rate starts “afresh” with each new harmonic stack. For example, if the principal block is replaced with a material which is 4 times lighter than the principal block, it creates a new harmonic stack with those 4 blocks plus 3 additional overhang steps.

The test which is documented here starts with blocks made of lead which have a density of 11g/cm^3 . The principal block of this 4-block harmonic stack is replaced with a concrete block, which has a density approximately a quarter of that of lead: 2.4g/cm^3 . The principal block of concrete is replaced by 4 blocks of wood, which have a density of 0.6g/cm^3 . The principal wood block is replaced by 4 Styrofoam blocks, which have a density of 0.02g/cm^3 .

Fewer blocks per harmonic stack density create a flatter, more aggressive cantilever. The strategy allows for more cantilever distance per block as illustrated in Figure 9. At the same time, the approach needs larger block sizes to achieve the same cantilever as larger harmonic stacks, or more density changes, which leads quickly to the use of unrealistically light material.

Exchange principal block with new harmonic stack with equal center of gravity and weight as principal block and double base

The strategy described above leads quickly to the use of extremely light materials. In response to that tendency, in a next step, a new variation was tested. Here, at the change of density, lighter blocks rest on more than one heavier block.

This reduces the amount of “weight loss” at the density jumps. At the same time, it allows for more density jumps in a given range of practical material densities. By thickening the base, this strategy also starts to enclose space. Space-making and structural behaviors are directly linked here as shown in Figure 10

CENTER OF GRAVITY

This series of investigations looked into the structural and spatial potential of modifying the center of gravity within blocks.

Shifting center of gravity to the edge of block.

Shifting the center of gravity to the edge of the blocks allows for a significantly greater overhang of a harmonic stack within the top courses. The principal block extends extremely far as the center of gravity of the principal block can be at the edge of the lower supporting block. The combined center of gravity of the principal block and the supporting lower block is also leaning to the edge, so the resulting overhang is again larger than in a typical harmonic stack. That overhang advantage is clearly visible for the first four to six blocks as documented in Figure 11.

An additional assembly tested how the top 4–6 block-advantage could be repeated in a single structure. It does so by creating a 10 block stack, but with a new stack at course 4 which acts as a counterweight. By repeating that logic, more and more counterweight is placed at the bottom stacks, which then again can cantilever further as shown in Figure 12.

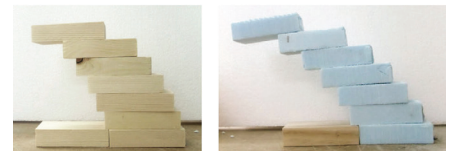
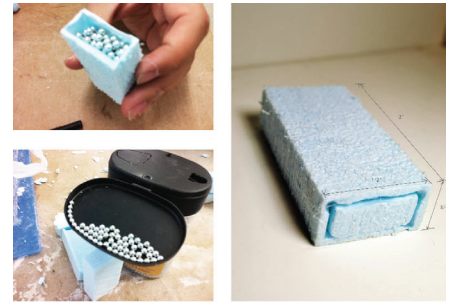
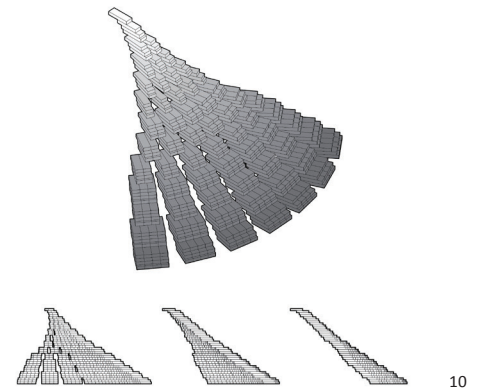
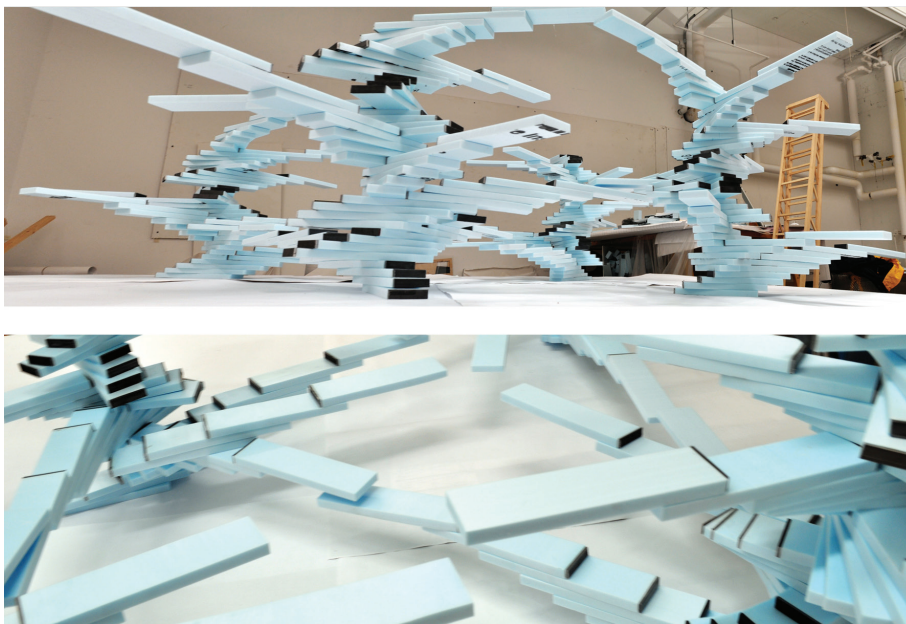


Figure 10: Extreme thickening of the base further reduces the amount of “weight loss” per density jump and increases the structural/spatial enclosure. A finely tuned weight loss per density change can lead to an extremely thickening base, enclosing space in a curved manner.

Figure 11: Comparison between a traditional harmonic stack (left) and a harmonic stack with shifted center of gravity (right). Blocks used in the model on the right were weighted along one edge.

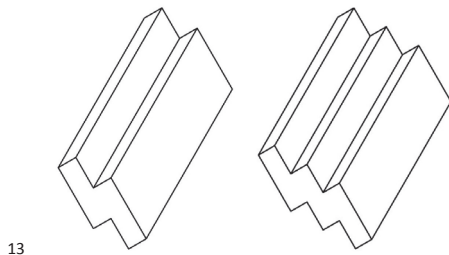
Figure 12: Physical test model, field of stacks. All models documented do not use any glue or bonding between blocks.



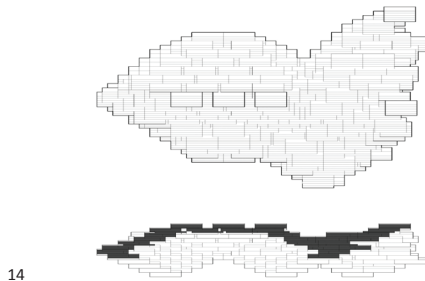
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12 blocks.



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BLOCK SHAPE

The block shape is also an important variable in corbelled stacks. This section documents two investigations into block shapes: an extruded saw tooth section and a non-directional circular disk.

Sawtooth cross section

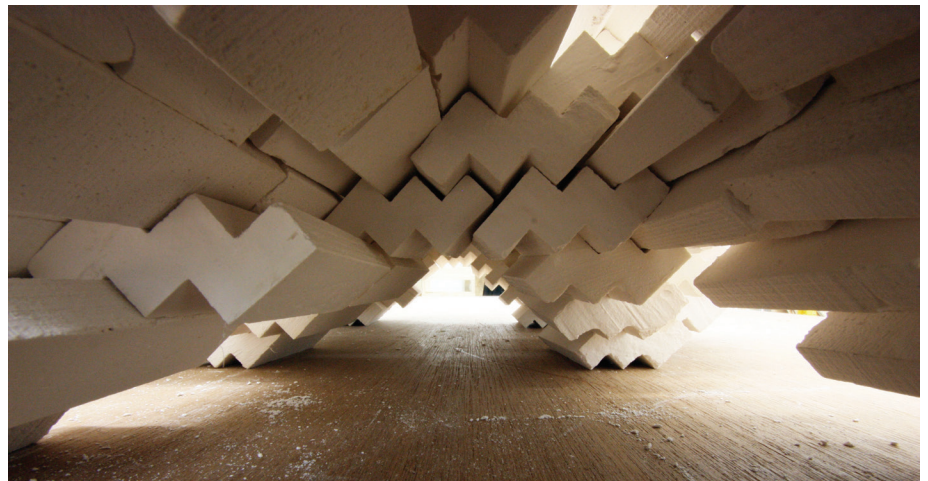
An extruded saw tooth block section was tested, to see how the interlocking geometry could increase overall stability and resistance against lateral forces. The simplest geometry is a two-toothed section. Stacking these blocks leads to leaning towers which are prone to tipping. A solution was to flip their orientation in each course. A three-toothed block typology was stable without flipping orientation (Figures 13 and 14).

The saw tooth shape was tested against optimal stack typologies. The lateral stability was significantly higher compared to a traditional optimal stack. Perpendicular to the stack orientation, blocks were completely secured against lateral movement. Pushing at the principal block tended to lead to a collapse of the whole structure.

The extruded saw tooth block shape is directional. Parallel to the block ridges, the block position can be adjusted smoothly and allows for optimal stack arrangement. Perpendicular to the block ridge lines blocks can only “jump” in increments of a tooth width. This characteristic creates interesting spatial effects, where the perception of space changes radically from parallel to perpendicular view direction as shown in Figure 15. Cross sections exhibit the optimal stack geometries; longitudinal sections show how the optimal stack sections are interlocked and form a unified structure.

Subsequent investigations looked at larger structures. Figure 14 show a canopy created by 264 blocks. The overall canopy rests on three points.

The saw tooth cross section allows for significantly more stable stacks against lateral forces compared to box-shaped blocks. An increase in vertical loads actually increases the overall stability of the structure and pushes blocks back into place. The interlocking saw tooth section performs similarly to mortar beds and comparable additive bonding agents.



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Figure 13: Two-toothed block (left) and three-toothed block (right).

Figure 14: Top view and section of a 264-block canopy resting on three “feet”.

Figure 15: Interior photo of a physical model of the 264-block canopy built with the extruded saw tooth block shape.

MATERIAL

The above investigations focused on the formal and structural implications and potentials of these new corbelled strategies. They were made with materials that were most simple to fabricate in large quantities. The current ongoing research focuses on applying our findings to construction grade materials. Two venues are tested, concrete and brick/ceramics. Figure 16 shows an initial test in concrete with a density gradient using a careful distribution

of aggregates. Further density gradient tests are being made with 3d-printed clay bricks. Experiments in slip cast ceramics were also undertaken, as shown in Figure 17.

CONCLUSIONS

Corbelled structures have been used since the Neolithic period. The construction of Newgrange in County Meath, Ireland, for example, dates back to 3200 BCE.⁸ There are two reasons for revisiting this age old technique: firstly, within the current discourse on sustainability, structures with long lifespans are ideal because they avoid potentially unnecessary construction waste and offer a high thermal mass. Secondly, new mathematical research radically questions standard assumptions about form and structural capabilities of corbelled structures. At this point, it seems important to revisit corbelled constructions because of their ability to be dry stacked, avoid mortar, avoid tensile forces and hybrid materialities like steel reinforced concrete. Disagreements in physical behaviors between different materials like steel and concrete are avoided altogether. This research took the papers “Overhang” and “Maximum overhang” as the basis for structural and spatial experimentation into corbelled structures. “Overhang” offers the most comprehensive investigation and groundbreaking rereading of corbelled structures and harmonic stacks so far. The aim of this research is to investigate the consequences and potentials for 3 dimensional structures and spatial typologies in architecture. The outcome so far suggests that there is a broad wealth of new typologies of corbelled structures in architecture which are, until now, not yet exhaustively researched or applied. There are two main challenges that corbelled structures pose. Firstly the high mass-to-void ratio requires a large amount of construction material. Secondly, stacking vast amount of blocks in a complex pattern requires significant effort. These issues need to be explored further in both their constraints but also in light of emerging technical advancements in automated fabrication. The research so far tested basic typologies in rather small scales and substitute materials. In our current research, construction grade materials like concrete will be tested using numerically controlled construction site robotic systems for the assembly of these complex structures.

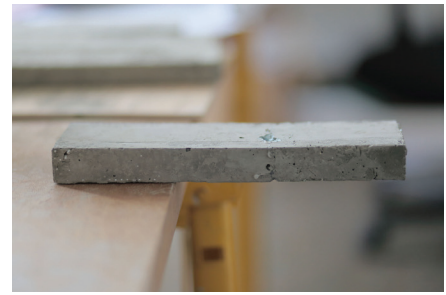
CREDITS

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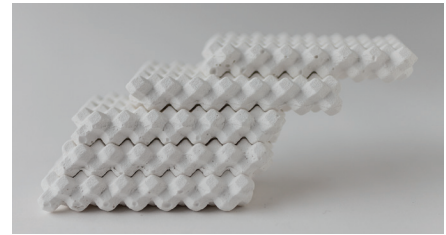
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Figure 16: Concrete block with inherent gradient of density allowing for further corbelling.

Figure 17: Slip cast ceramic blok tests.

ENDNOTES

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